Terminating and Recurring Decimal Expansions

Class 9 Mathematics · Chapter: Number Systems
🎬 **Video explanation (PrayogX reel):** https://www.instagram.com/reel/DfFuKmhj7E4/

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1. The puzzle

1 ÷ 4 = 0.25 and the division stops. But 1 ÷ 3 = 0.333… and the 3s never end. The reason is hidden in the **remainders** of long division.

2. Key definitions

Term	Meaning
Terminating decimal	A decimal expansion that ends after a finite number of digits, e.g. 1/4 = 0.25, 7/8 = 0.875.
Non-terminating recurring decimal	A decimal expansion that never ends but in which a block of digits repeats, e.g. 1/3 = 0.333…, 1/7 = 0.142857142857….
Non-terminating non-recurring decimal	Never ends and never repeats, e.g. $\sqrt{2}$ = 1.41421356…, π = 3.14159…; such numbers are irrational.
Bar notation	A bar over the repeating block: 0.142857142857… = $0.\overline{142857}$.

3. Why decimals stop or repeat

- At every step of long division a **remainder** is left.
- If the remainder becomes **0**, the division stops: the decimal **terminates**.
- The remainder is always **less than the divisor**, so dividing by q there are only q possible remainders (0 to q − 1). If 0 never comes, some remainder must **come back**, and from then on the digits repeat.

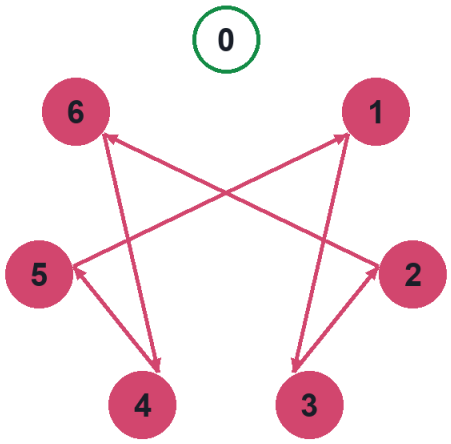
4. Worked out: 1 ÷ 7

Long division of 1 ÷ 7: watch the remainders

Step			
1	10 ÷ 7	1	3
2	30 ÷ 7	4	2
3	20 ÷ 7	2	6
4	60 ÷ 7	8	4
5	40 ÷ 7	5	5
6	50 ÷ 7	7	1
7	10 ÷ 7	1	3

1/7 = 0.142857142857… The remainder 3 came back, so the digits 142857 repeat for ever.

Remainders for ÷ 7 can only be 0–6: one must come back



Remainder 0 → the decimal stops (terminating). 0 never comes for 1 ÷ 7 → it repeats.

Remainders 3, 2, 6, 4, 5, 1 and then 3 again, so **1/7 = 0.142857** (a block of 6 digits, never more than 7 − 1 = 6).

5. The shortcut (for p/q in lowest form)

If the denominator q has no prime factors other than 2 and 5, the decimal terminates; otherwise it is non-terminating recurring.

Fraction	Denominator factors	Decimal
3/12 = 1/4	4 = 2 × 2	0.25 (terminating)
7/20	20 = 2 × 2 × 5	0.35 (terminating)
1/6	6 = 2 × 3	0.1666… = 0.16 (recurring)

Fraction	Denominator factors	Decimal
2/11	11	0.18 (recurring)

Always reduce to lowest form first: 3/12 has a 3 in its denominator, but 3/12 = 1/4 terminates.

6. Solved examples

Example 1. Find the decimal expansion of 3/8. $8 = 2^3$, so it terminates: 3/8 = **0.375**.
Example 2. Express 0.6 as p/q. Let x = 0.666…; 10x = 6.666…; 10x − x = 6, so 9x = 6 and x = **2/3**.
Example 3. Express 0.27 as p/q. Let x = 0.2727…; 100x = 27.2727…; 99x = 27, so x = 27/99 = **3/11**.

7. Key takeaways

- Every rational number has a decimal expansion that is either **terminating** or **non-terminating recurring**.
- A decimal that neither terminates nor repeats belongs to an **irrational** number.
- Remainders are always smaller than the divisor, which is why repetition is unavoidable when 0 never comes.

Practice questions (CBSE style)

1. Write the decimal expansion of 7/8 and state its type. (1 mark)
2. Find the decimal expansion of 1/11 and write it with bar notation. (2 marks)
3. Without dividing, state whether 13/125 and 5/6 have terminating or recurring expansions. Give reasons. (2 marks)
4. Express 0.4 in the form p/q. (2 marks)
5. Express 0.35 in the form p/q. (3 marks)
6. What can the maximum number of digits in the repeating block of 1/17 be? Why? (2 marks)
7. Classify as rational or irrational: 0.272727…, 0.101001000100001…, 2.5. (2 marks)

Answer key with worked solutions

1. 7/8 = 0.875, terminating ($8 = 2^3$).
2. 1/11 = 0.0909… = 0.09 (non-terminating recurring).
3. 13/125: $125 = 5^3$, only 5s → terminating (0.104). 5/6: 6 = 2 × 3 has the factor 3 → non-terminating recurring (0.8333…).
4. x = 0.444…; 10x = 4.444…; 9x = 4 → x = 4/9.
5. x = 0.3535…; 100x = 35.3535…; 99x = 35 → x = 35/99.
6. At most 16 digits, because the possible non-zero remainders when dividing by 17 are 1 to 16, so a remainder must repeat within 16 steps. (In fact the block has exactly 16 digits.)
7. 0.272727… is recurring → rational (3/11). 0.101001000100001… neither terminates nor repeats → irrational. 2.5 terminates → rational (5/2).