Why Division by Zero Is Undefined

PrayogX Maths · Supplementary topic
🎬 **Video explanation (PrayogX reel):** https://www.instagram.com/reel/DfFuzafE0qm/

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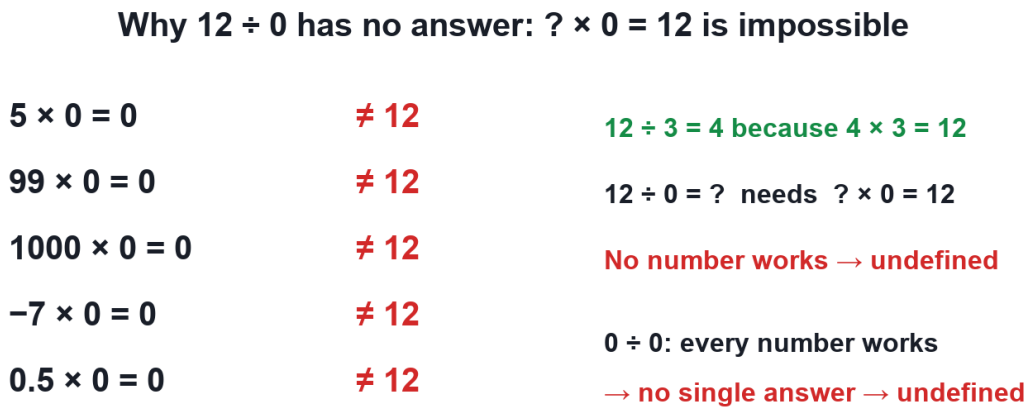
1. The calculator says ERROR

Type 12 ÷ 0 on any calculator and it shows **Error**. The calculator is not confused: there is **no number** that can be the answer.

2. Key definitions

Term	Meaning
Division as inverse of multiplication	a ÷ b = c means c × b = a (for b ≠ 0). Example: 12 ÷ 3 = 4 because 4 × 3 = 12.
Undefined	An expression that is not given any value because no number (or no single number) satisfies it.
Infinity (∞)	An idea of growing without bound; it is not a real number and cannot be used as an answer to a division.

3. Why a ÷ 0 (a ≠ 0) has no answer

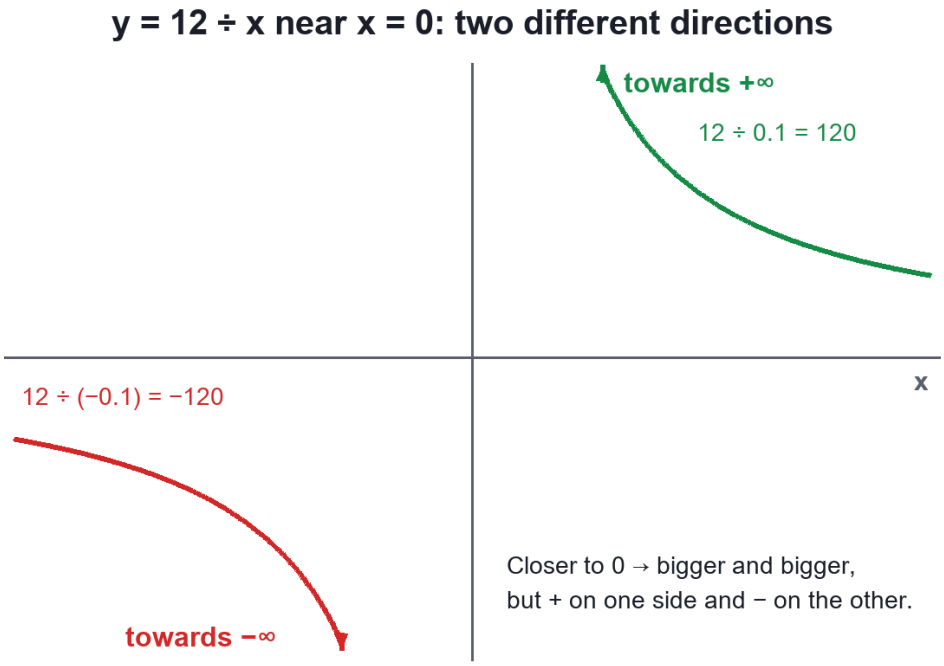


If 12 ÷ 0 = c, then c × 0 = 12. But **any number × 0 = 0**, never 12. So no c exists: 12 ÷ 0 is **undefined**.

4. Why 0 ÷ 0 is also undefined

If 0 ÷ 0 = c, then c × 0 = 0, which is true for **every** c (1, 5, −7, 1000…). There is no single answer, so 0 ÷ 0 is also not defined (it is called indeterminate).

5. Why the answer is not "infinity"



Divide 12 by smaller and smaller numbers: 12 ÷ 0.1 = 120, 12 ÷ 0.01 = 1200, 12 ÷ 0.001 = 12000 (heading to +∞). From the negative side: 12 ÷ (−0.01) = −1200 (heading to −∞). The two sides go in **opposite directions**, and infinity is not a number anyway, so 12 ÷ 0 has no value.

6. What IS allowed

Expression	Value	Reason
0 ÷ 5	0	0 × 5 = 0
5 × 0	0	Zero groups of 5
5 ÷ 0	Undefined	No c with c × 0 = 5
0 ÷ 0	Undefined	Every c works; no single answer

This is why a rational number p/q needs **q ≠ 0**.

7. Solved examples

Example 1. For which value of x is 5/(x − 3) undefined? When x − 3 = 0, i.e. **x = 3**.
Example 2. Is 0/7 a rational number? Yes: 0/7 = 0, and the denominator 7 is not zero.
Example 3. A student "proves" 1 = 2: let a = b; then $a^2 - b^2 = ab - b^2$, so (a + b)(a − b) = b(a − b); cancelling (a − b) gives a + b = b, so 2b = b and 2 = 1. The mistake: a − b = 0, so cancelling (a − b) is **dividing by zero**.

8. Key takeaways

- Division is the reverse of multiplication: a ÷ b = c means c × b = a.
- a ÷ 0 (a ≠ 0): no number works → undefined.
- 0 ÷ 0: every number works → no single answer → undefined.
- Infinity is not a number, so "12 ÷ 0 = ∞" is wrong.

Practice questions

1. Why is the denominator q in a rational number p/q never 0? (1 mark)
2. Find the value, if defined: (a) 0 ÷ 9 (b) 9 ÷ 0 (c) 0 × 9. (2 marks)
3. Explain using multiplication why 7 ÷ 0 is undefined. (2 marks)
4. For what value of x is (2x + 1)/(x + 4) not defined? (2 marks)
5. Why is 0 ÷ 0 undefined even though 0 × 5 = 0 is true? (2 marks)
6. Find the error in the "proof" that 1 = 2 given in Example 3. (3 marks)

Answer key with worked solutions

1. Because division by zero is undefined, so p/q would have no value if q = 0.
2. (a) 0 (b) undefined (c) 0.
3. If 7 ÷ 0 = c, then c × 0 = 7; but every number multiplied by 0 gives 0, never 7, so no such c exists.
4. Denominator x + 4 = 0 → x = −4.
5. If 0 ÷ 0 = c then c × 0 = 0, which is true for 5 and also for every other number, so there is no single value; hence undefined.
6. Since a = b, a − b = 0. Cancelling (a − b) from both sides divides both sides by zero, which is not allowed, so the step from (a + b)(a − b) = b(a − b) to a + b = b is invalid.