Equivalent Fractions and Comparison of Rational Numbers

Class 9 Mathematics · Chapter: Number Systems
🎬 **Video explanation (PrayogX reel):** https://www.instagram.com/reel/DfCxI9ZFTba/

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1. The pizza question

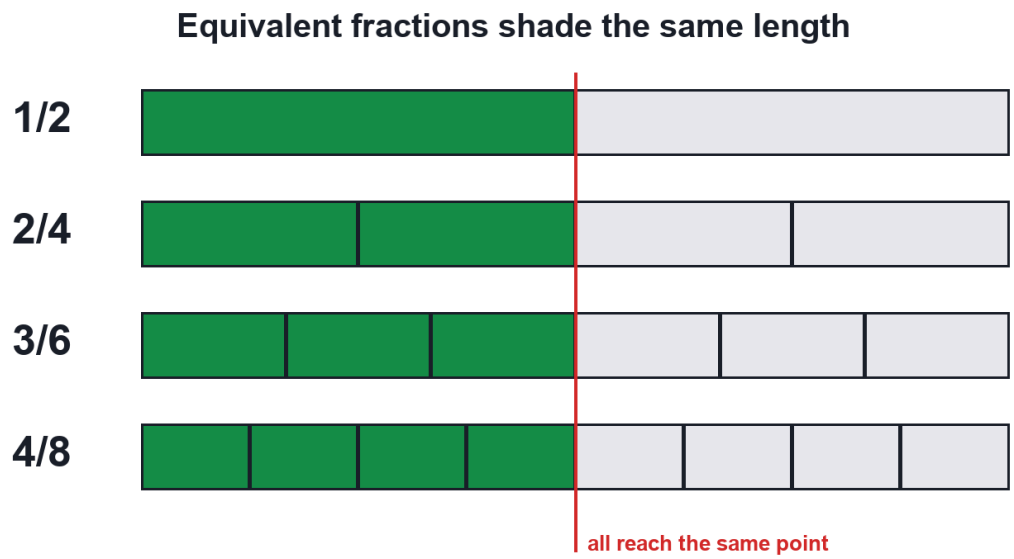
Rahul eats **1 of 2** slices of a pizza. Priya eats **4 of 8** slices of the same-size pizza. Who ate more? **Neither**: 1/2 and 4/8 are the same amount. They are **equivalent fractions**.

2. Key definitions

Term	Meaning
Rational number	A number that can be written as p/q, where p and q are integers and q ≠ 0 (e.g. 3/4, −5/6, 7 = 7/1).
Numerator / denominator	In p/q, p is the numerator (top), q the denominator (bottom).
Equivalent fractions	Fractions that represent the same value, e.g. 1/2 = 2/4 = 3/6 = 4/8.
Standard (lowest) form	p/q with q > 0 and HCF(p, q) = 1, e.g. 4/8 in standard form is 1/2.
LCM	Least common multiple: the smallest number that is a multiple of each given number.

3. The rule for equivalent fractions

p/q = (p × m)/(q × m) = (p ÷ m)/(q ÷ m), for any non-zero integer m (dividing only by a common factor).
Multiplying or dividing the numerator and the denominator by the **same non-zero number** does not change the value.

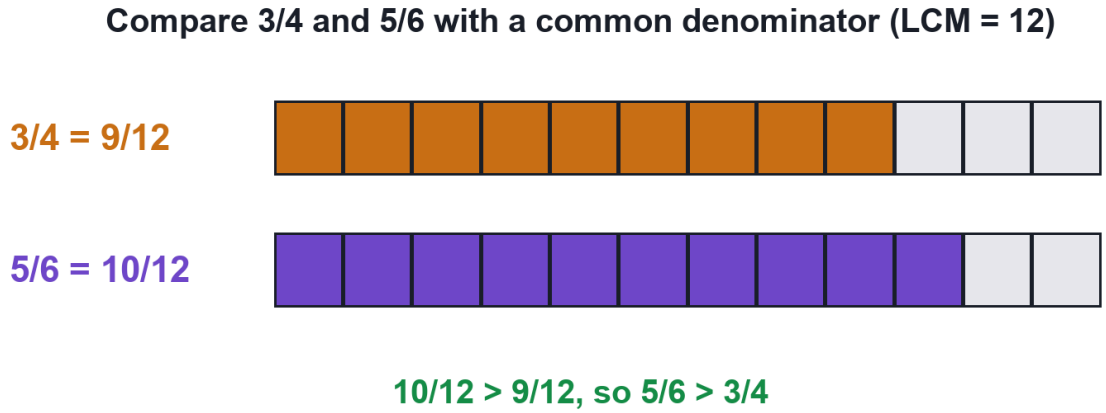


- 1/2 = (1×2)/(2×2) = 2/4 = (1×3)/(2×3) = 3/6 = (1×4)/(2×4) = 4/8
- Negative too: −3/5 = (−3×2)/(5×2) = −6/10
- **Test for equivalence (cross-multiplication):** a/b = c/d if and only if a × d = b × c. Example: 2/3 and 10/15 → 2 × 15 = 30 = 3 × 10 ✔

4. Comparing two rational numbers: step by step

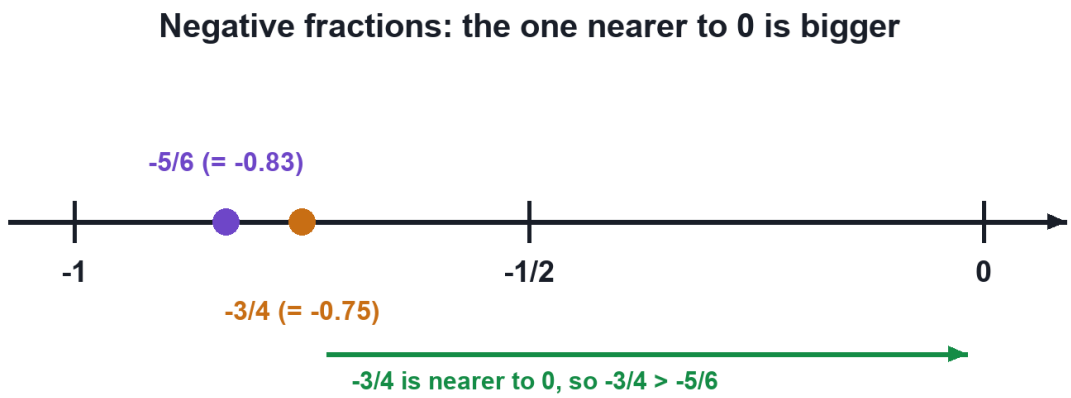
1. Write both numbers with a **positive denominator**.
2. Find the **LCM** of the denominators.
3. Convert each number to an equivalent fraction with the LCM as denominator.
4. Compare the **numerators**: the larger numerator gives the larger number.

Example: 3/4 vs 5/6. LCM(4, 6) = 12. 3/4 = 9/12 and 5/6 = 10/12. Since 10 > 9, **5/6 > 3/4**.



Shortcut (cross-multiplication): for a/b and c/d with b, d > 0, compare a × d with b × c. For 3/4 vs 5/6: 3 × 6 = 18 and 4 × 5 = 20; 18 < 20, so 3/4 < 5/6.

5. The negative twist



For negative numbers the order reverses: if 3/4 < 5/6, then −3/4 > −5/6. On the number line, −3/4 (= −0.75) lies to the **right** of −5/6 (≈ −0.83), nearer to 0, so it is bigger.
Using the LCM: −3/4 = −9/12 and −5/6 = −10/12; −9 > −10, so −9/12 > −10/12.
Remember: every positive rational number is greater than 0, and every negative one is less than 0; so any positive number > any negative number.

6. Analogies

- **Pizza / chocolate bar:** cutting the same pizza into more slices and taking proportionally more slices gives the same amount (1/2 = 4/8).
- **Negative numbers as temperatures:** −3 °C is warmer (greater) than −5 °C. In the same way −3/4 is greater than −5/6.

7. Solved examples

Solved example 1. Write three rational numbers equivalent to −2/7.
Solution: Multiply top and bottom by 2, 3, 4: **−4/14, −6/21, −8/28**.
Solved example 2. Which is greater: 7/9 or 4/5?
Solution: LCM(9, 5) = 45. 7/9 = 35/45, 4/5 = 36/45. 36 > 35, so **4/5 > 7/9**.
Solved example 3. Arrange in ascending order: −1/2, 2/3, −3/4, 1/6.
Solution: LCM(2, 3, 4, 6) = 12: −6/12, 8/12, −9/12, 2/12. Ascending order of numerators: −9, −6, 2, 8. So **−3/4 < −1/2 < 1/6 < 2/3**.

8. Common mistakes

- Adding the same number to top and bottom does **not** give an equivalent fraction (1/2 ≠ 2/3).
- Comparing only denominators: 3/4 vs 5/6 cannot be decided by "6 is bigger" alone.
- Forgetting that the order reverses for negative numbers.

9. Key takeaways

- Equivalent fractions: multiply or divide numerator and denominator by the same non-zero number.

- Compare by making denominators equal (LCM) and then comparing numerators.
- Cross-multiplication: a/b vs c/d → compare ad and bc (b, d > 0).
- For negatives the order reverses: −3/4 > −5/6 (nearer to 0).

Practice questions (CBSE style)

Q1 (MCQ). Which fraction is equivalent to 3/5?
(a) 6/15 (b) 9/15 (c) 5/3 (d) 4/6
Q2 (MCQ). Which is the greatest?
(a) −1/2 (b) −2/3 (c) −3/4 (d) −5/6
Q3 (Short answer, 2 marks). Write −12/18 in standard form and give two other rational numbers equivalent to it.
Q4 (Short answer, 2 marks). Check whether 5/8 and 15/24 are equivalent using cross-multiplication.
Q5 (Numerical, 3 marks). Compare 5/12 and 7/18 using the LCM method.
Q6 (Numerical, 3 marks). Arrange in descending order: 2/3, −5/9, 7/12, −1/6.
Q7 (Application, 3 marks). In a test, Aman scored 18 out of 24 and Sara scored 25 out of 30. Who did better? Show your working.
Q8 (Diagram based, 3 marks). Mark −3/4 and −5/6 on a number line between −1 and 0 and state which is greater.
Q9 (Case based, 4 marks). A baker cuts one cake into 6 equal pieces and an identical cake into 9 equal pieces. Ritu takes 4 pieces of the first cake, and Arjun takes 6 pieces of the second.
(a) Write the fraction of cake each one took. (1)
(b) Are the fractions equivalent? Show why. (1)
(c) Write both in standard form. (1)
(d) If Arjun had taken 7 pieces, who would have more? (1)

Answer key with worked solutions

A1. (b) 9/15 (= (3×3)/(5×3)).
A2. (a) −1/2. As decimals: −0.5, −0.67, −0.75, −0.83; −0.5 is nearest to 0.
A3. HCF(12, 18) = 6: −12/18 = **−2/3**. Equivalent numbers: −4/6, −6/9 (any two correct).
A4. 5 × 24 = 120 and 8 × 15 = 120. Equal products, so **yes, they are equivalent**.
A5. LCM(12, 18) = 36. 5/12 = 15/36, 7/18 = 14/36. 15 > 14, so **5/12 > 7/18**.
A6. LCM(3, 9, 12, 6) = 36: 2/3 = 24/36, −5/9 = −20/36, 7/12 = 21/36, −1/6 = −6/36. Descending: 24, 21, −6, −20 → **2/3 > 7/12 > −1/6 > −5/9**.
A7. Aman: 18/24 = 3/4 = 0.75; Sara: 25/30 = 5/6 ≈ 0.83. With LCM 12: 9/12 vs 10/12. **Sara did better**.
A8. Divide 0 to −1 into 12 equal parts: −3/4 = −9/12 (9 parts left of 0), −5/6 = −10/12 (10 parts left of 0). −3/4 is nearer to 0, so **−3/4 > −5/6**.
A9. (a) Ritu 4/6, Arjun 6/9. (b) 4 × 9 = 36 = 6 × 6, so yes, equivalent. (c) Both are **2/3**. (d) Arjun: 7/9 = 14/18 vs Ritu 4/6 = 12/18; **Arjun** would have more.