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Relations and Functions: Chapter Introduction

1. Relation and function

A **relation** R from a set A to a set B is any subset of A × B: any rule that pairs elements of A with elements of B.

A **function** f: A → B is a relation in which **every element of A has exactly one image** in B.

Pairing	Function?
student → roll number	Yes
student → birthday	Yes (each has exactly one)
student → friends	No (many outputs)

2. Types of functions

Type	Condition	Example
One-one (injective)	$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$	student → roll number; f(x) = 2x + 3
Many-one	two inputs share an image	student → birthday; f(x) = x² on ℝ
Onto (surjective)	every element of B is an image	f(x) = 2x + 3 on ℝ
Bijective	one-one and onto	f(x) = 2x + 3 on ℝ

Invertible: f has an inverse f^{-1} if and only if f is bijective.

Solved example. f(x) = 2x + 3 on ℝ. One-one: $2x_1 + 3 = 2x_2 + 3 \Rightarrow x_1 = x_2$. Onto: for any y, x = (y − 3)/2 gives f(x) = y. So $f^{-1}(y) = (y - 3)/2$.

3. Types of relations (on a set A)

- **Empty**: no pair; **universal**: every pair A × A.
- **Reflexive**: (a, a) ∈ R for all a.
- **Symmetric**: (a, b) ∈ R ⇒ (b, a) ∈ R.
- **Transitive**: (a, b), (b, c) ∈ R ⇒ (a, c) ∈ R.
- **Equivalence**: reflexive + symmetric + transitive, e.g. "is in the same class as".

4. Composition

For f: A → B and g: B → C, $(g \circ f)(x) = g(f(x))$. Example: f(x) = x + 1, g(x) = x² ⇒ $(g \circ f)(2) = g(3) = 9$.

Key takeaways

- Function = exactly one output per input.
- One-one: no two inputs share an output; onto: every output used.
- Only bijective functions are invertible.
- Equivalence = reflexive + symmetric + transitive.

Practice questions (CBSE style)

1. (MCQ) f: $\mathbb{R} \to \mathbb{R}$, $f(x) = x^2$ is (a) one-one (b) onto (c) many-one (d) bijective
2. (MCQ) A function is invertible iff it is (a) one-one (b) onto (c) bijective (d) many-one
3. (Very short) Is student → birthday one-one? Why?
4. (Very short) Find $(g \circ f)(2)$ for $f(x) = x + 1$, $g(x) = x^2$.
5. (Short) Show that "is in the same class as" is an equivalence relation.
6. (Numerical) Find f^{-1} for $f(x) = 2x + 3$.
7. (Numerical) Is f: $\mathbb{N} \to \mathbb{N}$, $f(n) = 2n$ onto? Justify.
8. (Application) A = {1, 2, 3}, R = {(1,1), (2,2), (3,3), (1,2), (2,1)}. Is R an equivalence relation?

Answer key

1. (c): $f(2) = f(-2) = 4$.
2. (c).
3. No: two students can share a birthday (many-one).
4. $g(f(2)) = g(3) = 9$.
5. Everyone is in their own class (reflexive); if a is in b's class, b is in a's (symmetric); if a, b and b, c share a class, so do a, c (transitive).
6. $y = 2x + 3 \Rightarrow x = (y - 3)/2$, so $f^{-1}(y) = (y - 3)/2$.
7. No: 1 (and every odd number) is not an image; f is one-one but not onto.
8. Yes: reflexive, symmetric ((1,2) and (2,1)), and transitive (every chain gives a pair already in R).

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