Why 0.999… Equals 1

Maths · Number Systems (supplementary) · PrayogX short notes

▸ **Video explanation:** Watch the PrayogX reel on Instagram

In one line: 0.999… (the 9s never stop) is **exactly** 1, not "a little less". They are two names for the same number.

Key definitions

Term	Meaning
Recurring (repeating) decimal	A decimal in which a digit or group of digits repeats forever, e.g. 0.333…, 0.999…, 0.142857142857…
Bar notation	0.999… is also written 0.9 with a bar over the 9.

Proof 1: using one third

1/3 = 0.333…
Multiply both sides by 3:
3 × 1/3 = 3 × 0.333…

1 = 0.999…

Proof 2: using algebra

Let x = 0.999…
Then 10x = 9.999…
Subtract: 10x − x = 9.999… − 0.999… (the endless 9s cancel)
9x = 9

x = 1

Proof 3: no number in between

- Between any two **different** numbers there is always another number (for example, their average).

- Try to find a number between 0.999… and 1: there is none, because any gap would show up at some decimal place, and the 9s never stop.
- No gap means they are the same number.

Related facts

- The same trick works for any recurring decimal: 0.777… = 7/9, and 0.454545… = 45/99 = 5/11.
- Every terminating decimal has a recurring-9 twin: 0.5 = 0.4999…, and 2.3 = 2.2999…
- Recurring decimals are rational numbers (they can be written as p/q).

Common mistakes

- "0.999… is the closest number to 1." There is no closest number below 1; 0.999… is 1.
- "The 9s stop somewhere far away." They never stop; 0.999 with three 9s is less than 1, but 0.999… is not.

Quick check

1. Write 0.444… as a fraction. *x = 0.444…, 10x = 4.444…, 9x = 4, so x = 4/9.*
2. Is 0.999 (exactly three 9s) equal to 1? *No, it is 1 − 0.001. Only the never-ending 0.999… equals 1.*

Takeaway

0.999… and 1: one number, two names.

▸ **Video explanation:** Watch the PrayogX reel on Instagram