Why 0.999… Equals 1

0.999… with the 9s never stopping is exactly 1, not "a little less". Three short proofs show why.

Name: Date: Score: ____ / 18

A. Fill in the blanks

1. 1/3 = 0.333…, so 3 × 1/3 = ____.

2. If x = 0.999…, then 10x − x = ____.

3. So 9x = 9 and x = ____.

4. 0.999 with three 9s is ____ than 1.

B. True or False

5. 0.999… is slightly less than 1. True / False

6. 0.999 (three 9s) equals 1. True / False

7. 0.999… and 1 are two names for the same number. True / False

8. 2/3 = 0.666… . True / False

C. Assertion and reason

9. Assertion (A): 0.999… = 1.
Reason (R): There is no number between 0.999… and 1.

(a) Both A and R are true, and R explains A
(b) Both A and R are true, but R does not explain A
(c) A is true, R is false
(d) A is false, R is true

Answer: _____

10. Assertion (A): 0.99 = 1.
Reason (R): 0.99 has only two 9s.

(a) Both A and R are true, and R explains A
(b) Both A and R are true, but R does not explain A
(c) A is true, R is false
(d) A is false, R is true

Answer: _____

D. Choose the correct option

11. x = 0.999…; 10x =

(a) 9.99…
(b) 9.999…
(c) 99.9
(d) 10

Answer: _____

12. 10x − x equals:

(a) 0.999…
(b) 9
(c) 10
(d) 1

Answer: _____

13. Which equals 1?

(a) 0.9 (b) 0.99

(c) 0.999 (d) 0.999…

Answer: _____

14. 0.4999… equals:

(a) 0.4 (b) 0.5

(c) 0.49 (d) 0.45

Answer: _____

E. Numerical problems

15. Using the x method, write 0.777… as a fraction.

F. Short answer questions

16. Prove that 0.999… = 1.

17. Use 1/3 = 0.333… to show 0.999… = 1.

G. Match the columns

18. Match each item in Column A with the correct item in Column B.

Column A	Column B	Your answer
1. 0.333…	(a) x = 1	1 → ____
2. 0.999…	(b) Less than 1	2 → ____
3. 0.999 (three 9s)	(c) 1	3 → ____
4. 10x − x = 9	(d) 1/3	4 → ____

Detailed solutions

1. 0.999…

2. 9

3. 1

4. less

5. False. The never-ending 0.999… equals 1 exactly.

6. False. It is 0.001 less than 1.

7. True. No number lies between them.

8. True. 2 × 0.333… .

9. (a) Two numbers with nothing between them are equal.

10. (d) A is false; R is true and is why 0.99 < 1.

11. (b) 9.999…. Shift the decimal point.

12. (b) 9. 9.999… − 0.999… = 9.

13. (d) 0.999…. Only the endless one.

14. (b) 0.5. Same reasoning.

15. Let x = 0.777…; 10x = 7.777…; 10x − x = 7, so 9x = 7; x = 7/9. **Answer: 7/9**

16. Let x = 0.999…. Then 10x = 9.999…. Subtract: 9x = 9, so x = 1.

17. Multiply by 3: 3 × 1/3 = 1 and 3 × 0.333… = 0.999…, so they are equal.

18. 1 → (d), 2 → (c), 3 → (b), 4 → (a). Endless repeating 9s equal the next number.